Microwaves

Lecture Notes

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These lecture notes are heavily based on the book of *Microwave Engineering* by David M. Pozar. For future versions or any proposals, please contact with *Dr. Serkan Aksoy* (saksoy@gyte.edu.tr).

Content

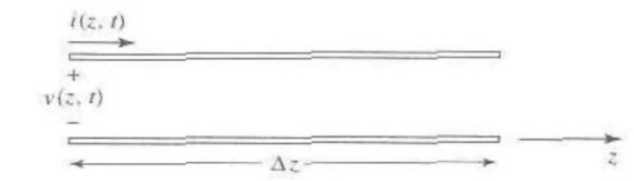
1. LUMPED CIRCUIT MODEL

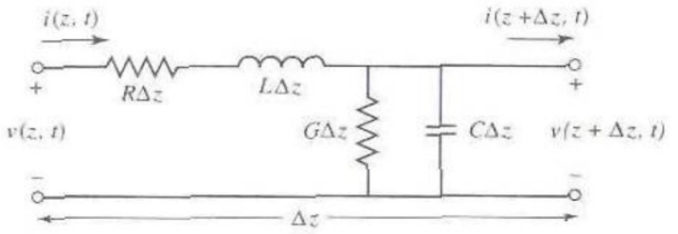
Fields $\rightarrow$ Circuits

L: Real size of the circuit, λ: Wavelength

$L \ll \lambda \quad \rightarrow \quad$ Circuit
$L = n\lambda\ (\lambda/n) \quad \rightarrow \quad$ Transmission Line

v and i will vary in magnitude and phase over its length.





R: Resistance per unit length (Ω/m): Finite conductivity
L: Inductance per unit length (H/m): Self inductance of two wires
C: Capacitance per unit length (F/m): Proximity conductors
G: Conductance per unit length (S/m): Dielectric losses

Kirchoff' Voltage Law

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

Kirchoff' Current Law

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Taking the limit as $\Delta z \rightarrow 0$, Telegraph equations

Time domain

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Frequency domain

$$\frac{dv(z)}{dz} = -(R + j\omega L)i(z)$$

$$\frac{di(z)}{dz} = -(G + j\omega C)v(z)$$

2. FIELD ANALYSIS of LINES

The idea is to find relations between $\vec{E}$ and $\vec{H}$ with R, L, G, C.

$$W_e = C\frac{|V_0|^2}{4} = \frac{\varepsilon}{4}\int_S \vec{E}\vec{E}^* ds \implies C = \frac{\varepsilon}{|V_0|^2}\int_S \vec{E}\vec{E}^* ds$$

$$W_m = L\frac{|I_0|^2}{4} = \frac{\mu}{4}\int_S \vec{H}\vec{H}^* ds \implies L = \frac{\mu}{|I_0|^2}\int_S \vec{H}\vec{H}^* ds$$

$$P_d = G\frac{|V_0|^2}{2} = \frac{\omega\varepsilon''}{2}\int_S \vec{E}\vec{E}^* ds \implies G = \frac{\omega\varepsilon''}{|V_0|^2}\int_S \vec{E}\vec{E}^* ds$$

$$P_c = R\frac{|I_0|^2}{2} = \frac{R_s}{2}\int_{C_1+C_2} \vec{H}\vec{H}^* dl \implies R = \frac{R_s}{|I_0|^2}\int_{C_1+C_2} \vec{H}\vec{H}^* dl$$

where $v_0 = V_0 e^{\mp jkz}$, $i_0 = I_0 e^{\pm jkz}$ and W_e, W_m, P_d, P_c are *time-averaged* values calculated for 1 m length line and S is the line cross-sectional area.

2.1. Wave Propagation along the Line

Using the frequency domain Telegrapher equation

$$\frac{d^2 v(z)}{dz^2} - \gamma^2 v(z) = 0 \quad , \quad \frac{d^2 i(z)}{dz^2} - \gamma^2 i(z) = 0$$

where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ is propagation constant. The solution of Telegrapher equation

$$v(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$i(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

Then, taking the z derivation of $v(z)$, the calculation of $i(z)$

$$i(z) = \frac{\gamma}{R + j\omega L}(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}) = \frac{v(z)}{Z(z)}$$

$$\Rightarrow \quad Z(z) = Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Then

$$i(z) = \frac{V_0^+}{Z}e^{-\gamma z} - \frac{V_0^-}{Z}e^{+\gamma z}$$

Converting to time domain by using $\gamma = \alpha + jk$

$$v(z,t) = |V_0^+| cos(\omega t - kz + \emptyset^+)e^{-\alpha z}$$

$$+|V_0^-| cos(\omega t - kz + \emptyset^-)e^{\alpha z}$$

where $\lambda = 2\pi/k$, $v_{phase} = \omega/k = \lambda f$, $\emptyset^{\mp}$ is the phase.

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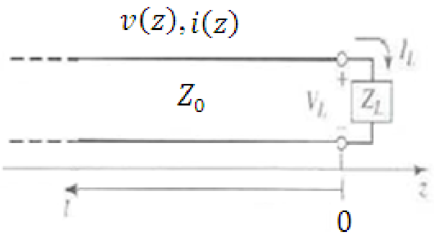
2.1.1. Lossless Transmission Lines

$$R = G = 0 \;\Rightarrow\; \gamma = \alpha + jk \;\Rightarrow\; \alpha = 0\,, k = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \lambda = \frac{2\pi}{j\omega\sqrt{LC}} \qquad v_{phase} = \frac{1}{\sqrt{LC}}$$

When the lossless line is terminated by a load Z_L



$Z_L \neq Z_0 \;\Longrightarrow$ Reflected waves occur.

$$v(z) = V_0^+ e^{-jkz} + V_0^- e^{+jkz} \;\Rightarrow\; i(z) = \frac{V_0^+}{Z} e^{-jkz} - \frac{V_0^-}{Z} e^{+jkz}$$

Reflection coefficient at the load, $z = 0$

$$Z_L = \frac{v(0)}{i(0)} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \;\Longrightarrow\; \Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$v(z) = V_0^+ \left(e^{-jkz} + \Gamma e^{+jkz}\right) \;\Rightarrow\; i(z) = \frac{V_0^+}{Z}\left(e^{-jkz} - \Gamma e^{+jkz}\right)$$

where $v(z)$ and $i(z)$ consists of a superposition of an incident and reflected waves called *Standing Waves* ($\Gamma \neq 0$).

Time Average Power Flow:

$$P_{av} = \frac{1}{2} Re\{v(z) i^*(z)\} = \frac{|V_0^+|^2}{2Z}\left(1 - |\Gamma|^2\right)$$

This shows P_{av} is constant at anywhere on the line.

When the line is matched. $Z_L = Z_0 \;\Longrightarrow\; \Gamma = 0$.

$|v(z)| = V_0^+$ is constant.

• When the line is mismatched ($Z_L \neq Z_0$, $\Gamma \neq 0$). *Return Loss*

$$\text{RL} = -20 log|\Gamma|$$

RL = 0 ($-\infty$ dB) $\Longrightarrow$ $\Gamma = 0$ Matched load (No reflected power, maximum power is delivered).
RL = 1 (0 dB) $\Longrightarrow$ $|\Gamma| = 1$ Total reflection, (All power reflected).

$$|v(z)| = |V_0^+| \overbrace{\left|1 + \Gamma e^{j2kz}\right|}^{|1+|\Gamma| e^{j(\theta - 2kl)}|} \neq |V_0^+| \text{ is not constant.}$$

The maximum value $V_{max} = |V_0^+|(1 + |\Gamma|)$ occurs when $e^{j(\theta - 2kl)} = 1$. The minimum value $V_{min} = |V_0^+|(1 - |\Gamma|)$ occurs when $e^{j(\theta - 2kl)} = -1$. When Γ increases, V_{max}/V_{min} increases as a measure of mismatch. Then Standing Wave Ratio ($1 \leq SWR \leq \infty$) is

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

when $SWR = 1$ means matched line. In that case at $z = -l$, the reflection coefficient and input impedance

$$\Gamma(l) = \frac{V_0^- e^{-jkl}}{V_0^+ e^{+jkl}} = \Gamma(0) e^{-j2kl}$$

$$Z_{in} = \frac{v(-l)}{i(-l)} = Z_0 \frac{V_0^+\left[e^{jkl} + \Gamma e^{-jkl}\right]}{V_0^+\left[e^{jkl} - \Gamma e^{-jkl}\right]} = Z_0 \frac{1 + \Gamma e^{-j2kl}}{1 - \Gamma e^{-j2kl}}$$

Using the definition of Γ, more useful form known as *Transmission Line Impedance Equation* as

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 tan(kl)}{Z_0 + jZ_L tan(kl)}$$

- Transmission Coefficient:Some part of EM wave is also transmitted to second region as

$$V_0^+\left[e^{-jkz} + \Gamma e^{+jkz}\right]\Big|_{z=0} = V_0^+ \mathrm{T} e^{-jkz}\Big|_{z=0}$$

$$\mathrm{T} = \frac{V_{transmitted}}{V^+} = 1 + \Gamma = \frac{2Z_L}{Z_L + Z_0}$$

Insertion Loss: IL $= -20 log|\mathrm{T}|$ $[dB]$

• Short Circuit:
$Z_L = 0 \;\Longrightarrow\; \Gamma = -1$, SWR $\rightarrow \infty$, $Z_{in}^{short} = jZ_0 tan(kl)$

• Open Circuit:
$Z_L = \infty \;\Longrightarrow\; \Gamma = +1$, SWR $\rightarrow \infty$, $Z_{in}^{open} = -jZ_0 cot(kl)$

$$Z_0 = \sqrt{Z_{in}^{short} Z_{in}^{open}}$$

The proper length of open or short circuited transmission line can provide any desired reactance or susceptance.

• $l = \lambda/2 \Longrightarrow Z_{in} = Z_L$ (No regard to Z_0): The same impedance is observed at the input.

• $l = \lambda/4 + n\,\lambda/2 \Longrightarrow Z_{in} = Z_0^2/Z_L$ (Quarter Wave Transform)
 - $Z_L = 0$ (Short circuit) $\Longrightarrow$ $Z_{in} \rightarrow \infty$ Open circuit
 - $Z_L = \infty$ (Open circuit) $\Longrightarrow$ $Z_{in} \rightarrow 0$. Short circuit

2.1.2. Lossy Transmission Lines

In practice, finite conductivity (or lossy dielectrics) lines can be evaluated as a *Lossy Line*.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$

Low-loss line $R \ll \omega L$, $G \ll \omega C \Rightarrow RG \ll \omega^2 LC$. Then ignoring the last term of $\sqrt{1 \mp x} \cong 1 \mp x/2 \pm x^2/8 \ldots$

- $\gamma \cong j\omega\sqrt{LC}\sqrt{1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)} = \alpha + jk$
- $\alpha \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$, $k \cong \omega\sqrt{LC}$
- $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \cong \sqrt{\frac{L}{C}}$, $Z_0^{coax} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\, ln\left(\frac{b}{a}\right)$

In the lossy line; α, k & Z_0 can be approximated to lossless line.

- *Distortionless Line*: For the lossy line, in fact the exact $k \neq \omega\sqrt{LC}$ is not a linear function of frequency means dispersive. But specifically if the following condition holds

$$\frac{R}{L} = \frac{G}{C}$$

then $\alpha = R\sqrt{C/L}$, $k = \omega\sqrt{LC}$ mean that the lossy line behaves as lossless (distortionless) line.

- *Terminated Lossy Line*: Loss is assumed small that $Z \cong Z_0$

$$Z_{in} = \frac{v(-l)}{i(-l)} = Z_0\frac{Z_L + Z_0 tanh(\gamma l)}{Z_0 + Z_L tanh(\gamma l)}$$

$$P_{in}|_{z=-l} = \frac{1}{2}Re\{v(-l)i^*(-l)\} = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2)e^{2\alpha l}$$

$$P_{load}|_{z=0} = \frac{1}{2}Re\{v(0)i^*(0)\} = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2)$$

Power lost in the line:

$$P_{loss} = P_{in} - P_{load} = \frac{|V_0^+|^2}{2Z_0}[(e^{2\alpha l} - 1) + |\Gamma|^2(1 - e^{-2\alpha l})]$$

- Perturbation Method for Calculating Attenuation

Power flow along lossy line: $P(z) = P_0 e^{-2\alpha z}$

Power loss per unit length: $P_{loss}^{unit} \triangleq -\frac{\partial P(z)}{\partial z} = \overbrace{2\alpha P_0 e^{-2\alpha z}}^{2\alpha P(z)}$

Attenuation constant: $\alpha = \frac{P_{loss}^{unit}}{2P(z)} = \frac{P_{loss}^{unit}(z=0)}{2P_0}$

- Wheeler Incremental Inductance Rule:

$L = \frac{\mu}{|I|^2}\int_S \left|\vec{H}\right|^2 ds$: Inductance per length

$\Delta L = \frac{\mu_0 \delta_S}{2|I|^2}\int_l \left|\vec{H}_t\right|^2 dl$: Incremental inductance

$P_{loss}^{unit} = \frac{R_S}{2}\int_l \left|\vec{H}_t\right|^2 dl = \frac{R_S |I|^2 \Delta L}{\mu_0 \delta_S} = \frac{|I|^2 \omega \Delta L}{2}$: Loss per unit length

$P_0 = |I|^2 \frac{Z_0}{2}$: Power at the $z = 0$.

Then, Attenuation constant:

$$\alpha = \frac{P_{loss}}{2P_0} = \frac{\omega \Delta L}{2Z_0} = \frac{k\Delta Z_0}{2Z_0}$$

where δ_S is skin depth and $Z_0 = \sqrt{L/C}$.

- Taylor series Inductance Rule:

$$\left.\begin{aligned} Z_0\left(\frac{\delta_S}{2}\right) &\cong Z_0 + \frac{\delta_S}{2}\frac{dZ_0}{dl} \\ \Delta Z_0 = Z_0\left(\frac{\delta_S}{2}\right) - Z_0 &= \frac{\delta_S}{2}\left(\frac{dZ_0}{dl}\right) \end{aligned}\right\} \Rightarrow \alpha = \frac{k\Delta Z_0}{2Z_0} = \frac{R_s}{2Z_0\eta}\frac{dZ_0}{dl}$$

where $= \sqrt{\mu_0/\varepsilon_0}$.

2.2. Smith Chart

$$\Gamma \leftrightarrow Z = R + jX \quad , \quad Y = G + jB$$

where R is Resistance, X is Reactance, G is Conductance and B is Susceptance. Whenever $z = Z/Z_0$ is normalized impedance

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma|e^{j\theta} \Rightarrow z_L = \frac{1 + |\Gamma|e^{j\theta}}{1 - |\Gamma|e^{j\theta}}$$

$$\Gamma = \Gamma_r + j\Gamma_i \quad ; \quad z_L = r_L + jx_L$$

The apsis and ordinate of Smith chart are Γ_i and Γ_r.

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad , \quad x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Rearranging them

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$
$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2$$

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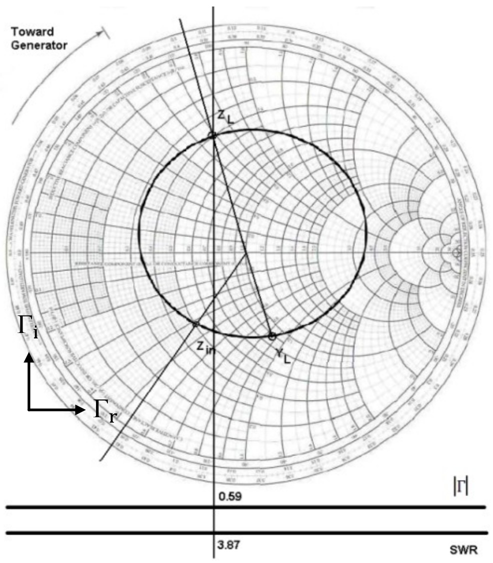
These are two families of circles as r_L and x_L. Superposition of Smith Chart and its 180° ($\lambda/4$) rotated version is known as *Combined Impedance-Admittance Smith Chart*.



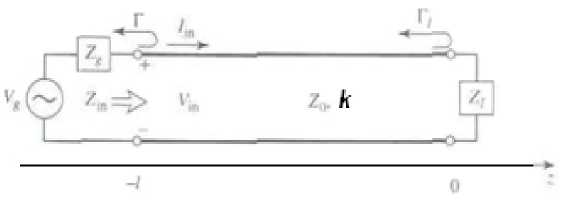
$\lambda/2$ is the complete revolution of Smith chart. $\lambda/4$ is the half of Smith chart (180°). The images of Z is Y in Smith chart.

2.3. Slotted Line

This device is used to find Z_L as first V_{min}.

- Measurement of SWR on the l_{min} distance from the line.
- Calculate $|\Gamma| = (SWR-1)/(SWR+1)$
- $e^{j(\theta-2kl)}\big|_{V_{min}} = -1 \implies \theta = \pi + 2kl_{min}$
- Using θ and $|\Gamma|$, write $\Gamma = |\Gamma|e^{j\theta}$
- Calculate $Z_L = Z_0[(1+\Gamma)/(1-\Gamma)]$ at $l = 0$.

2.4. Generator & Load Mismatches



$$v(z) = V_0^+ e^{-jkz} + V_0^- e^{+jkz} = V_0^+(e^{-jkz} + \Gamma_l e^{jkz})$$

$$v(-l) = V_0^+(e^{jkl} + \Gamma_l e^{-jkl}) = \frac{Z_{in}}{Z_{in}+Z_g}V_g$$

$$Z_{in} = Z_0 \frac{1+\Gamma_l e^{-j2kl}}{1-\Gamma_l e^{-j2kl}}\bigg|_{z=-l}$$

Then, using this

$$V_0^+ = \frac{Z_0}{Z_0+Z_g}V_g \frac{e^{-jkl}}{(1-\Gamma_l\Gamma_g e^{-j2kl})}$$

where $\Gamma_g = Z_g - Z_0/Z_g + Z_0$ and $\Gamma_l = Z_l - Z_0/Z_l + Z_0$.

$$P = \frac{1}{2}Re\{v_{in}i_{in}^*\} = \frac{|V_g|^2}{2}\frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}$$

where $Z_{in} = R_{in} + jX_{in}$ and $Z_g = R_g + jX_g$. Generally Z_g is fixed and three cases are considered as

• Load Matched to Line: $Z_l = Z_0$, $\Gamma_l = 0$, $SWR = 1$. Then $Z_{in} = Z_0$.

$$P = \frac{|V_g|^2}{2}\frac{Z_0}{(Z_0+R_g)^2 + X_g^2}$$

• Generator Matched to Line: $Z_{in} = Z_g$, $\Gamma_g = 0$.

$$P = \frac{|V_g|^2}{2}\frac{R_g}{4(R_g^2+X_g^2)}$$

• Conjugate Latching: $Z_{in} = Z_g^*$

$$\frac{\partial P}{\partial R_{in}} = 0 \implies \left.\begin{matrix} R_{in} = R_g \\ X_{in} = -X_g \end{matrix}\right\} \implies Z_{in} = Z_g^*$$

$$P = \frac{|V_g|^2}{2}\frac{1}{4R_g^2}$$

Maximum power transfer $\Gamma = \Gamma_g = \Gamma_l = 0$. If one directly chose $\Gamma = \Gamma_g = \Gamma_l = 0$, it does not mean that the best efficiency due to the phase differences. The efficiency can be improved only by making Z_g as small as possible.

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3. MICROWAVE NETWORKS

At low frequencies for electrically small circuits, lumped active and passive circuit elements are enough for analyzing the circuit leading a type of a Quasi-Static solution (assumption of negligible phase change in any where of the circuit) of Maxwell's equations and to Kirchoff Current and Voltage Laws with impedance concept. Moreover fields are considered as TEM type. But this way is not possible to analyze microwave circuits. The circuit concept should modify and apply to microwave network theory developed in MIT in 1940. The reasons of using it are as follow

- Much easier than field theory,
- Calculations are performed at terminals, not everywhere,
- Easy to modify and combine different problems,
- The field solution of Maxwell's equation gives more information at the every time and place of the network, but difficult. At microwave frequencies, although the definition of the terminal pairs for TEM line is relatively easy, the terminal pair for $non-TEM$ line does not strictly exist.

3.1. Voltage, Current and Impedance

The measurements of V and I at microwave frequencies are difficult due to not easily defined terminals for non-TEM waves. Because of that the fields are measured and used as

$$V = \int_{+}^{-} \vec{E}\vec{dl} \quad , \quad I = \oint_{C^+} \vec{H}\vec{dl}$$

than, the impedance can be defined as $Z = V/I$. Because the fields depend on the coordinates (like in waveguide), special attenuation should be given for extraction of V and I. The way is to do that

$$\vec{E} = \frac{e(x,y)}{a}\underbrace{(V^+e^{-jkz} + V^-e^{+jkz})}_{v(z)}$$

$$\vec{H} = \frac{h(x,y)}{b}\underbrace{(I^+e^{-jkz} + I^-e^{+jkz})}_{i(z)}$$

then, the impedance can be defined as $Z = V^+/I^+ = V^-/I^- = a/b$. The impedance concept first used by O. Heaviside, and then after application to transmission lines, to electromagnetics by Schelkunoff. In this manner, types of it

- Intrinsic Impedance, $Z = \sqrt{\mu/\varepsilon}$: It depends on only the material parameters.
- Wave Impedance, $Z = \vec{E}_{tang.}/\vec{H}_{tang.}$: TM, TE, TEM types are present which may depend on type of guide, material and frequency.
- Characteristic Impedance, $Z = \sqrt{L/C}$: For TEM, it is unique, but for TE and TM, not unique because v and i can not be determined, uniquely.

It can be shown that the real parts of $v(\omega)$, $i(\omega)$ and $Z(\omega)$ are even in ω, but the imaginery parts of them are odd in ω. $|\Gamma(\omega)|$ and $|\Gamma(\omega)|^2$ are the even function in ω.

3.2. Impedance & Admittance Matrices

The v and i of a N port microwave network having n'th terminal

$$v_n = V_n^+ + V_n^- \quad , \quad i_n = I_n^+ + I_n^-$$

The impedance matrix is in the form of $[v] = [Z][i]$

$$\begin{bmatrix} v_1 \\ v_2 \\ . \\ v_n \end{bmatrix} = \begin{bmatrix} Z_{11} & . & Z_{1N} \\ . & . & . \\ Z_{N1} & . & Z_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ . \\ i_n \end{bmatrix}$$

Similarly the admittance matrix is in the form of $[i] = [Y][v]$

$$\begin{bmatrix} i_1 \\ i_2 \\ . \\ i_n \end{bmatrix} = \begin{bmatrix} Y_{11} & . & Y_{1N} \\ . & . & . \\ Y_{N1} & . & Y_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ . \\ v_n \end{bmatrix}$$

Clear that $[Y] = [Z]^{-1}$. It can be shown that

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{Open\ circuiting\ all\ other\ ports}$$

$$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{Short\ circuiting\ all\ other\ ports}$$

Then Z_{ii} and Z_{ij} are known as *Input Impedance* and *Transfer Impedance*, respectively.

- If the network is reciprocal (no ferrites, plasmas and active devices inside), $Z_{ij} = Z_{ji}$ and $Y_{ij} = Y_{ji}$ are the right relations.
- If the network is lossless, Z_{ij} and Y_{ij} are purely imaginary.

3.2.1. Scattering Matrix

The form $[V^-] = [S][V^+]$ of the scattering matrix gives the complete description of the N port networks with the incident and reflected waves as

$$\begin{bmatrix} V_1^- \\ V_2^- \\ . \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & . & S_{1N} \\ . & . & . \\ S_{N1} & . & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ . \\ V_N^+ \end{bmatrix}$$

Any element of the scattering matrix

$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{All\ ports\ matched\ with\ loads}$$

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S_{ii} and S_{ij} are *Reflection* and *Transmission Coefficient* (from port j to port i), respectively. Network analyzer is used to measure S parameters of a network. If network is reciprocal, $[S]$ is symmetric. If network is lossless, $[S]$ is unitary means that $\sum_{k=1}^{N} S_{ki}S_{kj}^{*} = \delta_{ij}$. Using the relations as $[V] = [V^+] + [V^-]$ and $[I] = [I^+] - [I^-]$ with $Z_0 = 1$ between $[S]$ and $[Z]$ matrixes

$$[S] = ([Z] + [I])^{-1}([Z] - [I])$$

$$[Z] = ([I] + [S])([I] - [S])^{-1}$$

In the $[S]$, all the V_i and I_i are defined as a reference point at the end of every lines. If the reference point is shifted, then

$$S_{ii}^{shifted} = e^{-j2k_i l_i} S_{ii}$$

where $k_i l_i$ is the electrical length of the outward shift.

3.2.1.1. Generalized Scattering Matrix

In the previous chapter, $[S]$ is defined for networks with same characteristic impedance for all ports. Generally for not same impedance for all ports, a new set of wave amplitudes as

$$a_i = \frac{V_i^+}{\sqrt{Z_{0i}}} \quad , \quad b_i = \frac{V_i^-}{\sqrt{Z_{0i}}}$$

Then, generalized S matrix

$$S_{ij} = \left.\frac{b_i}{a_j}\right|_{\substack{All\ ports \\ matched \\ with\ loads}} = \left.\frac{V_i^- \sqrt{Z_{0i}}}{V_j^+ \sqrt{Z_{0j}}}\right|_{\substack{All\ ports \\ matched \\ with\ loads}}$$

- In reciprocal networks, $[S] = [S]^t$ is symmetric,
- In lossless networks, $[S]$ is unitary and satisfies the equation

$$\sum_{k=1}^{N} S_{ki}S_{kj}^{*} = \delta_{ij}$$

3.2.2. Transmission (ABCD) Matrix

Many microwave networks consisting cascade connection and need building block fashion in practice. ABCD matrix is defined to satisfy this as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

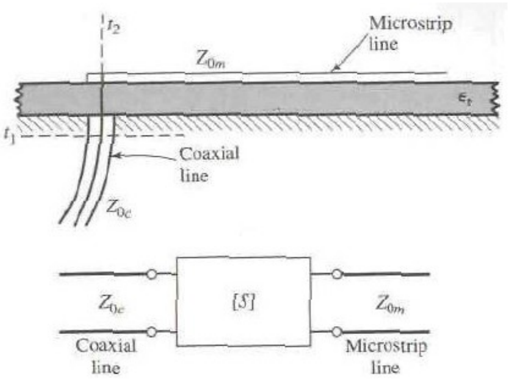
where port 1 and port 2 are completely isolated in the equation means that cascade multiplication is possible. The current direction is also specially designed for ABCD matrix. The relation between Z and $ABCD$ matrix parameters are as

$$A = \frac{Z_{11}}{Z_{12}} \quad , \quad B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} \quad , \quad C = \frac{1}{Z_{21}} \quad , \quad D = \frac{Z_{22}}{Z_{21}}$$

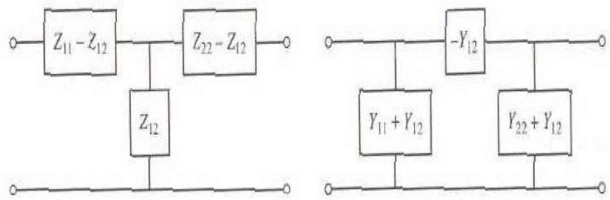
If the network is reciprocal ($Z_{12} = Z_{21}$), then $AD - BC = 1$.

3.3. Equivalent Circuits for 2 Port Networks

Transition between a coaxial line and microstrip line can be chosen as an example of two port networks. Because of discontinuity in the transition region, EM energy is stored in the vicinity of the transition leading to reactive effects mean that the transition region should be modeled as black box (There is an unlimited way of equivalent circuits, but choosing S matrix equivalence), then



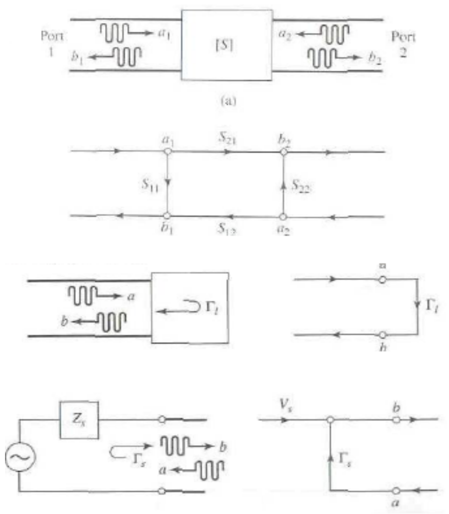
• A nonreciprocal network can not be represented by passive equivalent circuit using reciprocal elements. If the network is reciprocal (there are six degrees of freedom, six independent parameters), the presentations lead naturally to T and π equivalent circuits as



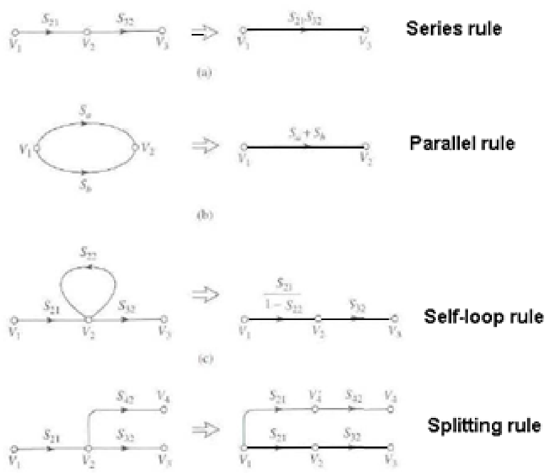
• If the network is lossless, the impedance and admittance matrix elements are purely imaginary, the degrees of freedom reduces to three and the elements of T and π equivalent circuits should be constructed from purely reactive elements.

3.4. Signal Flow Graphs

Signal flow graph is an additional technique to analyse microwave networks in terms of reflected and transmitted waves. Three different forms of it are given below with nodes and branches.



The decomposition rules are also given below



These lecture notes are heavily based on the book of *Microwave Engineering* by David M. Pozar. For future versions or any proposals, please contact with *Dr. Serkan Aksoy* (saksoy@gyte.edu.tr).

4. IMPEDANCE MATCHING

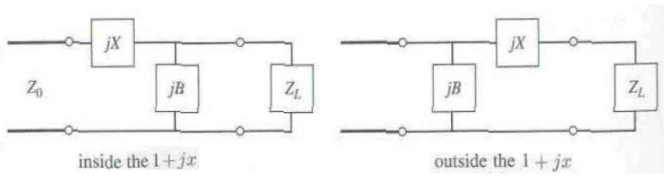
Impedance matching (or tuning) is an important issue for

- Maximum power is delivered when load is matched to line (assuming the generator is matched)
- Power loss is minimized.
- S/N ratio of receiver components is increased.
- Amplitude and phase errors are reduced.

Whenever Z_L has nonzero real part, impedance matching is possible with the factors such as

- Complexity: Simpler one is prefable.
- Wide Bandwidth: Match a load over a band.
- Implementation: Easier one is prefable
- Adjustability: Adjust to match a variable load impedance.

4.1. L Networks Matching

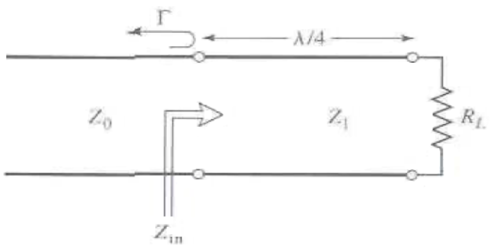


The normalized z_L should be converted to $1 + jx$ with adding $j\beta$ impedance, then adding $-jx$ the impedance matching will be successful.

$$\frac{\beta}{Z_0} = 2\pi f C \quad \Rightarrow \quad C = \frac{\beta}{2\pi f C\, Z_0}$$

$$xZ_0 = 2\pi f L \quad \Rightarrow \quad L = \frac{xZ_0}{2\pi f}$$

4.2. Quarter Wave Transformer



It is used for <u>only real load</u> impedance. Complex load impedance can always be transformed to real impedance by appropriate length of transmission line. But this generally alters the frequency dependency of the equivalent load reducing the bandwidth of the matching. The following relation has to be satisfied as

$$\Gamma = 0 \ \Rightarrow \ Z_{in} = Z_0$$

where

$$Z_{in} = Z_1 \frac{R_L + jZ_1 tan(kl)}{Z_1 + jR_L tan(kl)}$$

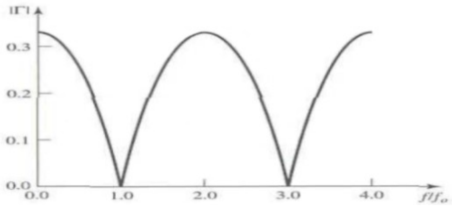
$$kl = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \ \Rightarrow \ Z_{in} = \frac{Z_1^2}{R_L} = Z_0 \ \Rightarrow \ Z_1 = \sqrt{Z_0 R_L}$$

For multiple reflection, total Γ is

$$\Gamma = \frac{\Gamma_1 + \Gamma_1\Gamma_2\Gamma_3 - T_1T_2\Gamma_3}{1 + \Gamma_2\Gamma_3} = \frac{2(Z_1^2 - Z_0 R_L)}{(Z_1 + Z_0)(R_L + Z_1)}$$

The condition of $\Gamma = 0 \Rightarrow Z_1^2 = Z_0 R_L \Rightarrow Z_1 = \sqrt{Z_0 R_L}$ is also enough to make total reflection (multiple) is zero.

If $Z_1 = \sqrt{Z_0 R_L}$ and $l = \lambda/4$, then $\Gamma = 0$ and $SWR = 1$.

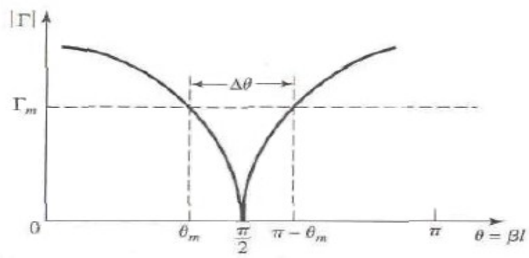


For each frequency, l has to be equal to $\lambda/4$. Thus fixed line length is possible only for one frequency.

- Bandwidth performance of QWT for wide band matching: If one set the maximum value of reflection coefficient, $\Gamma = \Gamma_m$ that can be tolerated, then the fractional bandwidth is

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} arcos\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.\right]$$

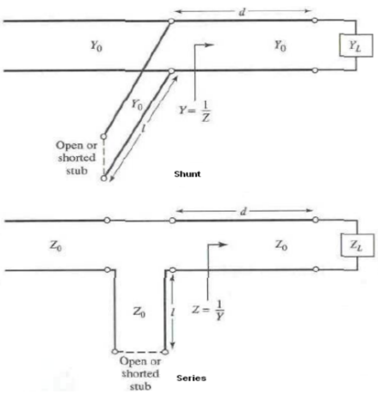
This shows that as Z_L becomes closer to Z_0 , the bandwidth increases. This result is valid only for TEM lines. The step changes of reactance effect can be compensated by making a small adjustment in the length of the matching section. Approximate behavior of reflection coefficients is shown at below.



The QWT can be extended as a multisection form for matching of broader bandwidth.

4.3. Single Stub Tuning

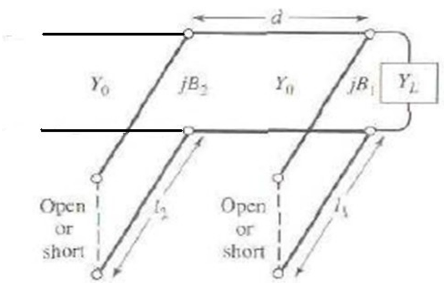
A single open-circuited (or short-circuited) transmission line is connected either in parallel or series with the feed line at a certain distance from the load. Single Sub Tuning <u>can match any load impedance</u> to line, but suffer from <u>disadvantage of requiring a variable length</u> between the load and stub.



Since lumped elements are not required, single stub is convenient and easy to fabricate in microstrip form. Two adjustable parameters d and susceptance or reactance provided by stub. Although for microstrip lines open circuit is easy to fabricate since a via-hole is enough, for coax or waveguide short circuit is preferred since open circuit line may be large for radiation. If the impedance has the form of $Z_0 + jX$ at the distance d. Then stub reactance can be chosen as $-jX$, resulting for matching condition. For a given susceptance or reactance, the difference in lengths of open and short-circuited stub is $\lambda/4$.

4.4. Double Stub Tuning

The double stub tuner can not match all load impedances, but load may be arbitrary distance from the first stub.



Distance between the stubs should be generally chosen as $\lambda/8$ or $3\lambda/8$ to reduce the frequency sensitivity.

• *Single Section Transformer*: The reflection coefficient of single section transformer can be written when discontinuities between the impedances are small as

$$\Gamma \cong \Gamma_1 + \Gamma_3 e^{-j2kl}$$

This shows that Γ is dominated by Γ_1 and Γ_3.

• *Multisection Transformer*: If the applications require more bandwidth, multisection transformers consists of N equal length sections of the lines can be used. The reflection coefficient can be written as

$$\Gamma(kl) \cong \Gamma_0 + \Gamma_1 e^{-j2kl} + \Gamma_2 e^{-j4kl} + \cdots + +\Gamma_N e^{-j2Nkl}$$

The importance of this result is that the desired reflection coefficients response as a function of frequency (kl) can be synthesized by proper choosing of Γ_N. To obtain passband responses, binominal (maximally flat) and Chebysev (equal ripple) multisection matching transformers can be used. In first one: the $N-1$ derivatives of $|\Gamma(\theta)|$ is settled to zero, in second one: $\Gamma(\theta)$ is equated to Chebyshev polynomial.

4.5. Tapered Lines

The line can be continuously tapered for decreasing the effect of the step changes in characteristic impedance between the discrete sections. The incremental reflection coefficient

$$d\Gamma = \lim_{\Delta z \to 0} \Delta\Gamma = \lim_{\Delta z \to 0} \frac{(Z+\Delta Z) - Z}{(Z+\Delta Z)+Z} \cong \lim_{\Delta z \to 0} \frac{1}{2}\frac{\Delta Z}{Z} = \frac{1}{2}\frac{dZ}{Z}$$

$$d\Gamma = \frac{1}{2}\frac{dZ}{dz} = \frac{1}{2}\frac{d(lnZ/Z_0)}{dz}dz$$

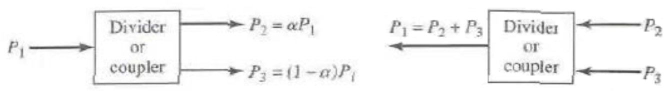
Since $d(lnf(z))/dz = (1/f)\,df(z)/dz$, by using theory of small reflections

$$\Gamma(\theta) = \frac{1}{2}\int_{z=0}^{L} e^{-j2kz}\frac{d}{dz}ln\left(\frac{Z(z)}{Z_0}\right)dz$$

where $\Gamma(\theta)$ from $Z(z)$ can be found. Chancing type of taper (Exponential, $Z(z) = e^{az}$, Triangular, Klopfenstein), different band pass characteristic may be applied. Klopfenstein yields the shortest matching section. The Bode-Fano criterion for certain type of canonical load impedances will help us to define theoretical limit on the minimum reflection with the upper limit of matching performance and provide a benchmark against which a practical design can be compared.

5. POWER DIVIDERS & COUPLERS

These are passive components used for power division or power combining. In power division, an input signal is divided by coupler in two (or more) signals, equally or not.



5.1. Three Port Networks (T Junction)

It has two inputs with one output.

• If component is passive (no anisotropic material), the network is reciprocal ($S_{ij} = S_{ji}$) and when all ports are also matched ($S_{ii} = 0$), considering lossless one

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

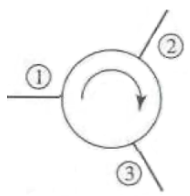
Using the features of $[S]$ for lossless and reciprocal network

$$|S_{12}|^2 + |S_{13}|^2 = 1 \, , |S_{12}|^2 + |S_{23}|^2 = 1 \, , |S_{13}|^2 + |S_{23}|^2 = 1$$

$$S_{13}^* S_{23} = 0 \, , S_{23}^* S_{12} = 0 \, , S_{12}^* S_{13} = 0$$

To satisfy, above equation at least two parameters have to be zero means that three port network can not be reciprocal, lossless and matched all ports.

• If the network is nonreciprocal with matching all port and satisfaction of energy conservation, such a device is known as *Circulator* relies on anisotropic materials.



$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

• if only two ports of the network are matched, a lossless and reciprocal network can be physically realizable.

• If the network is being lossy, network can be reciprocal and matched at all ports (*Resistive Divider* or *Isolator*). As an example, *Ferrite Isolators* are two-port device having unidirectional transmission characteristics. Because $[S]$ is not unitary, the isolator must be lossy. The isolators can be used between a high-power source and load to prevent possible reflections from damaging the source by absorbing reflected power.

5.1.1. T Junction Power Divider

This can be used for power division (or combining).

• *Lossless Divider*: This suffers from the problem of not being matched at all ports and in addition does not have any isolation between two output ports. The fringing fields and higher order modes at the discontinuity leading stored energy can be accounted by a lumped susceptance, B. The output line impedances Z_1 and Z_2 can be selected to provide various power decision.

• *Resistive Divider*: Possible to match all ports simultaneously, the resistive (lossy) divider is used, but no isolation between two output ports due to being not lossless. Half of the supplied power is dissipated in resistors.

5.2. Four Port Networks (Directional Coupler)

It has two inputs and two outputs. After considering using the features of $[S]$ matrix for reciprocal, matched and lossless network, the possible solutions are $S_{14} = S_{23} = 0$ means *Directional Coupler*. Using different phase references, *Symmetrical* or *Anti-symmetrical Directional Coupler* may be defined. The design parameters of directional coupler are

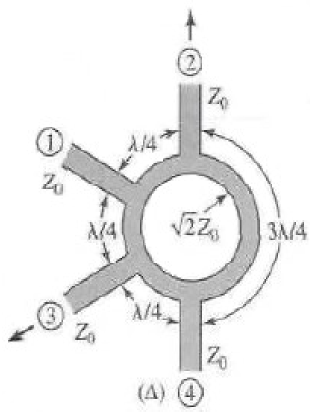
$$\left.\begin{aligned} Coupling &= C = -20log|S_{13}| \\ Directivity &= D = 20log\frac{|S_{13}|}{|S_{14}|} \\ Isolation &= I = -20log|S_{14}| \end{aligned}\right\} \Rightarrow I = D + C \; dB$$

The coupling factor shows the fraction of input power to the output. The directivity is a measure of isolation ability for forward and backward waves. The ideal coupler has infinite directivity and isolation and also lossless.

The directional property of the all directional coupler is produced through the use of two separate waves or wave components, which add in phase at the coupled port, and cancel in phase at the isolated port.

5.2.1. Waveguide Directional Coupler

• *Bethe Hole Coupler*: Couple one waveguide to another through a single small hole in the common wall. Types of the parallel guides and skewed guides work properly only at the design frequency (narrow bandwidth in terms of its directivity).



• *Multi Hole Coupler*: Series of coupling holes are used to increase bandwidth as similar design to multi section transformer. Making coupling coefficients proportional to binominal coefficients, maximally flat response can be obtained. Using Chebysev polynomial, different responses are possible.

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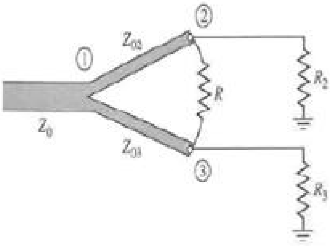
5.2.2. Wilkinson Power Divider



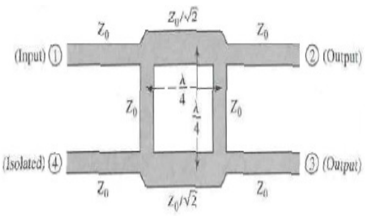
It is a network with the useful property of being lossless when the output ports are matched, that is, only reflected power is dissipated. It is known that a lossy three port network can be made having all ports are matched with isolation between the output ports. Wilkinson Power Divider can be made in microstrip or stripline form with arbitrary power division of N way Divider or Combiner. The even-odd mode technique is used for analysis.

5.2.3. Hybrid Coupler

It has $C = 3\ dB$ having types of the following.

5.2.3.1. Quadrature Hybrid (90° Hybrid)

This is a $3\,dB$ directional coupler (knows as *Branch Line Hybrid*) with a 90° phase difference in outputs ($2 \rightarrow 3$). Even-odd mode technique can be applied for analysis. $[S]$ matrix has a high degree of symmetry means any port can be used for input as given below



$$[S] = \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

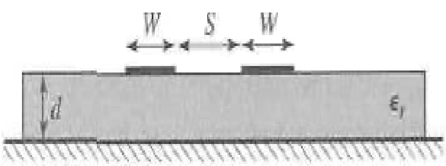
5.2.4. 180° Hybrid

It is a four port network with a 180° phase shift ($2 \rightarrow 3$) between two outputs (also may be in phase). It can be used as a combiner and has unitary symmetric scattering matrix as

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix}$$

It may be produces as the form of ring hybrid (rate race), tapered matching lines and hybrid waveguide junction (Magic T, (Rate Race)) in which symmetrically (or antisymmetrical) placed tuning ports (or irises) can be used for matching.

5.2.5. Coupled Line Directional Coupler



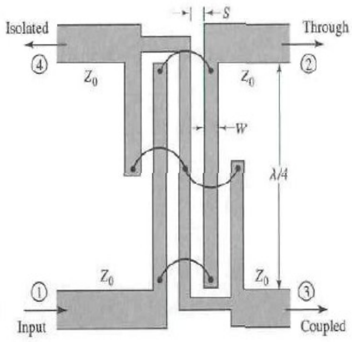
Coupled lines of two (or more) transmission lines are closed together, power can be coupled between the lines. Generally TEM mode is assumed rigorously valid for striplines, but approximately valid for microstrips. Coupled Line Theory is based on types of excitations as even mode (strip currents are equal in amplitude with same directions) and odd mode (strip currents are equal in amplitude with opposite directions). Arbitrary excitation can be treated as a superposition of appropriate even and odd modes amplitudes. Moreover design graphs are present for coupled lines.

- Design Considerations:

- Although a single section coupled line has limited bandwidth due to $\lambda/4$ requirement, the bandwidth can be increased using multiple sections coupled line having close relations to multisection QWT.

- The assumption of the same velocity of propagation for even and odd modes in design, generally not satisfied for a coupled microstrip or non TEM lines. This gives poor directivity. By using more effective dielectric constant (smaller phase velocity) for even mode, phase differences should be minimized. This also produces problems as the mismatching phase velocities for multisection case and degrades coupler directivity. Increasing bandwidth can be obtained with low coupling limits.

5.2.6. Lange Coupler



To increase coupling factor, Lange Coupler (several lines) with 90° phase difference between outputs is used as a 3 dB coupling ratio in an octave or more bandwidth can be achieved. The main disadvantage of it (a type of quadrature hybrid) is difficult to fabricate due to very narrow lines. Folded Lange coupler is also used for more easily analysis to model equivalent circuit.

5.3. Other Couplers

- Moreno Crossed Guide Coupler
- Schwinger Reversed Phase Coupler
- Riblet Short Slot Coupler
- Symmetric Tapered Coupled Line Coupler
- Coupler with Apertures in Planar Lines

As an example of a device uses a directional coupler is *Reflectometer* isolate and sample the incident and reflected powers from a mismatch load as a heart of a scalar (or vectorial) network analyzer.

6. NOISE & ACTIVE COMPONENTS

Noise is usually generated by random motions of charges (or charge carriers in devices and materials). Such motions can be caused by the mechanism of

- Thermal Noise: Thermal vibrations of bound charges.
- Shot Noise: Random fluctuations of charge carriers.
- Flicker Noise: $1/f$ noise.
- Plasma Noise: Random motions of charges.
- Quantum Noise: Quantized nature of charge carriers.

Noise is a random process and can be passed into a system from external sources or generated within the system itself. Noise level defining the system performance determines for minimum signal reliability detected by a receiver.

- *Dynamic Range and Compression Point*: The linearity and deterministic features of all components can be satisfied in a range called *Dynamic Range*. The floor level of noise dominates the output power at very low frequencies. $1\ dB$ *Compression Point* is defined as the input power for which the output is $1\ dB$ below that of an ideal amplifier.

- *Noise Power and Equivalent Noise Temperature*: Rayleigh-Jeans approximation results *Voltage Fluctuations* as

$$V_n = \sqrt{4kTBR}$$

where k is Boltzmann's constant, $T\ °K$ is temperature, B is bandwidth and R is resistance. Because of frequency independency, this is known as *White Noise Source* can be treated as Gaussian distributed variables.

A noisy resistor R can be replaced with a noiseless resistor and a voltage source of RMS V_n. Then connecting a load resistor R results in maximum power transfer called *Noise Power* as

$$P_n = \frac{V_n^2}{4R} = kTB$$

- $T \to 0$ then $P_n \to 0$: Cooler device, less noise power.
- $B \to 0$ then $P_n \to 0$: Smaller bandwidth, less noise power.
- $B \to \infty$ then $P_n \to \infty$: Use the exact definition of V_n for P_n.

If P_n is not strong function of frequency (*White Noise*), an *Equivalent Noise Temperature* is defined as

$$T_e = \frac{N_0}{kB}$$

where N_0 is *Noise Power* delivered to load R.

A noisy amplifier with a source of resistor at a temperature of $T = 0\ °K$ can be replaced with a noiseless amplifier and a resistor having Equivalent Noise Temperature T_e as

$$T_e = \frac{N_0}{GkB}$$

where N_0 is output noise power and G is amplifier gain. *Excess Noise Ratio (ENR)* is also used to characterize Noise Power of active noise generator consisting of a diode or a tube as

$$ENR = 10log\frac{N_g - N_0}{N_0} = 10log\frac{T_g - T_0}{T_0}$$

where N_g & T_g are Noise Power & Equivalent Temperature of generator.

- Measurement of Noise Power: Y factor method is applied as

$$\left.\begin{aligned} N_1 &= GkT_1B + GkT_eB \\ N_2 &= GkT_2B + GkT_eB \end{aligned}\right\} \Rightarrow Y = \frac{N_1}{N_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1$$

where Y should be determined via power measurement. Then

$$T_e = \frac{T_1 - YT_2}{Y - 1}$$

where T_1 & T_2 are temperature of hot & cold load, respectively.

6.1. Noise Figure, *F*

It is a measure of the degration in S/N ratio between the input and output as

$$F = \frac{S_i/N_i}{S_0/N_0} \geq 1$$

F is defined for a matched input source and for a noise source consist of a resistor temperature $T_i = 290\ °K$.

- *Noise Figure of a Noisy Network*: Having the parameters of (G, B, T_e) with input noise $N_i = kT_iB$ and signal power S_i, the output noise power $N_0 = kGB(T_i + T_e)$ and the output noise signal $S_0 = GS_i$. Then Noise Figure is

$$F = \frac{S_i/kT_iB}{GS_i/kGB(T_i + T_e)} = 1 + \frac{T_e}{T_i} \geq 1 \quad \Rightarrow \quad T_e = (F - 1)T_i$$

If the network is noiseless $(T_e = 0)$, $F = 1\ (0\ dB)$.

- *Noise Figure of a Two-Port Passive and Lossy Network*: Having $G < 1$ such as attenuator (or lossy line) with a matched source resistor at T, overall system temperature also at T, noise factor

$$N_0 = kTB = GkTB + GN_{line}$$

$$\Rightarrow N_{line} = \frac{1 - G}{G}kTB = (L - 1)kTB$$

where $L = 1/G$ is lossy factor. The equivalent noise temperature

$$T_e = \frac{1-G}{G}T = (L-1)T \;\Rightarrow\; F = 1 + \frac{T_e}{T_i} = 1 + (L-1)\frac{T}{T_i}$$

If the line is at the temperature T_i, $F = L$.

• *Noise Figure of Cascaded System*: A cascaded system of two components (G_1, F_1, T_{e1}), (G_2, F_2, T_{e2}) and $N_i = kT_iB$

The output of first stage : $N_1 = G_1kT_iB + G_1kT_{e1}B$
The output of second stage:

$$N_0 = G_2N_1 + G_2kT_{e2}B = G_1G_2kB\left(T_i + T_{e1} + \frac{1}{G_1}T_{e2}\right)$$

$$T_{cascade} = T_{e1} + \frac{1}{G_1}T_{e2}$$

$$F_{cascade} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

where $T_{e1} = (F_1 - 1)T_i$ and $T_{e2} = (F_2 - 1)T_i$. The first stage is dominant due to $1/G_1$ reduces to second stage. Generally

$$T_{cascade} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2}$$

$$F_{cascade} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2}$$

• *Noise Figure of a Passive Two Port Network*: *Available Power Gain* G_{12} does not depend on Γ_L.

$$G_{12} = \frac{Power\ avaliable\ from\ Network}{Power\ available\ from\ Source} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1 - S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}$$

where $\Gamma_{out} = S_{22} + (S_{12}S_{21}\Gamma_S/(1 - S_{11}\Gamma_S))$, then $N_2 = kTB$ is as

$$N_2 = G_{21}kTB + G_{21}N_{network} \;\Rightarrow\; N_{network} = \frac{1-G_{21}}{G_{21}}kTB$$

$$T_e = \frac{N_{network}}{kB} = \frac{1-G_{21}}{G_{21}}T \;\Rightarrow\; F = 1 + \frac{T_e}{T_i} = 1 + \frac{1-G_{21}}{G_{21}}\frac{T}{T_i}$$

The definitions of T_e and F are close as lossy lines. When the network is matched $\Gamma_S = 0$, $S_{22} = 0$, $\Gamma_{out} = 0$, then $G_{12} = |S_{21}|^2$.

• *Noise Figure of a Mismatched Lossy Line*: Previously, Noise Figure of a lossy line is calculated under assumptions of line is matched to its input and output circuits. Consider that line (L, T, Z_0, B) is mismatched to input circuit as

$$\Gamma_S = \frac{Z_g - Z_0}{Z_g + Z_0} \;\Rightarrow\; [S] = \frac{e^{-jkl}}{\sqrt{L}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Then

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = \frac{\Gamma_S}{L}e^{-j2kl} \;\Rightarrow\; G_{12} = \frac{L(1-|\Gamma_S|^2)}{L^2 - |\Gamma_S|^2}$$

$$T_e = \frac{1-G_{21}}{G_{21}}T = \frac{(L-1)(L+|\Gamma_S|^2)}{L(1-|\Gamma_S|^2)}T$$

When the line is matched $\Gamma_S = 0 \;\Rightarrow\; T_e = (L-1)T$ same as the matched lossy line.

6.2. Dynamic Range & Intermodulation Distortion

All realistic devices are nonlinear at very low power levels due to noise effects and also practical components became nonlinear at high power levels. A given component (or network) can operate as desired between minimum and maximum realistic power ranges known as *Dynamic Range*. In the sense of intermodulation distortion, it is called as *Spurious Free Dynamic Range*. The output response of a nonlinear device (diode, transistor) by using a Taylor series expansion

$$v_0 = a_0 + a_1v_i + a_2v_i^2 + a_3v_i^3 + \cdots$$

where $a_0 = v_0(0)$ is DC output, $a_1 = dv_0/dv_i|_{v_i=0}$ is Lineer output, others are Nonlinear outputs. Different output response can be obtained relating to coefficients as

Only $a_0 \neq 0$, Rectifier
Only $a_1 \neq 0$, Attenuator or Amplifier
Only $a_2 \neq 0$, Mixing or Frequency Conversion

• *Gain Compression:* $v_i = V_0cos(\omega_0t)$

$$G = a_1 + \frac{3}{4}a_3V_0^2$$

In most practical system, $a_3 < 0$, then gain tends to decrease named *Gain Compression* or *Saturation*.

• *Intermodulation Distortion:* $v_i = V_0[cos(\omega_1t) + cos\omega 2t$

$$G_v = m\omega_1 + n\omega_2$$

$m, n = 0, \pm1, \pm2, \ldots$ the output is the combination of two input frequencies are called *Intermodulation Products*.

Passive Intermodulation: Connectors, cables, antennas, and every metal-metal contact can cause passive intermodulation due to poor mechanical contact, oxidation, contamination etc., and also thermal effects of high power source. This has generally lower power levels.

6.3. RF Diode Characteristics

• *Shottky Barrier Diode Detectors*: This is a nonlinear device consisting of semiconductor-metal junction resulting lower junction capacitance can be used frequency conversion (rectification, detection, mixing). It has a

$$I(V) = I_S\left(e^{\frac{q}{nkT}V} - 1\right)$$

with a Small Signal Model

$$I(V) = I_0 + vG_d + \frac{v^2}{2}G'_d + \cdots$$

These diodes are used as rectifiers, detectors and demodulation of an AM modulated RF carrier.

• *PIN Diode:* This is used to construct an electronic switching for control circuits such as phase shifters and attenuators. These are preferable because of small size, high speed and inerrability with planar circuits. Especially single-pole PIN diode switches can be used in either a series or a shunt configuration to form a single pole RF switch. Insertion Loss of switches

$$IL_{series} = -20log\left(\frac{2Z_0}{2Z_0 + Z_d}\right) \ , \ IL_{shunt} = -20log\left(\frac{2Z_d}{2Z_d + Z_0}\right)$$

where Z_d is diode impedance as

$$Z_d^{reverse} = R_r + j\left(\omega L - \frac{1}{\omega C}\right) \ , \ Z_d^{forward} = R_r + j\omega L$$

• *Varactor Diode*: Junction capacitance varies with bias voltage used for electronically frequency tuning.

• *Impatt Diode*: Similar to PIN diode, but based on avalanche effects exhibiting negative resistance over a broad frequency range, therefore used to directly convert DC to RF power.

• *Gunn Diode*: It exhibits a negative differential resistance based on Gunn effect and used to generate RF power to DC.

• *Baritt Diode*: Similar to junction transistor without a base contact and useful for detector and mixer applications with advantages of lower AM noise.

7. MICROWAVE AMPLIFIER DESIGN

7.1. Two-Port Power Gains

The gain and stability of a general two-port amplifier in terms of S parameters of transistor will be investigated for amplifier and oscillator design. Three types of power gain can be derived as

$$Power\ Gain, G = \frac{P_L}{P_{in}} = \frac{Power\ dissipated\ in\ Z_L}{Power\ delivered\ to\ input}$$

$$Available\ Gain, G_A = \frac{P_{avn}}{P_{avs}} = \frac{Power\ available\ from\ network}{Power\ dissipated\ from\ source}$$

$$Transduser\ Gain, G_T = \frac{P_L}{P_{avs}} = \frac{Power\ delivered\ to\ Z_L}{Power\ available\ from\ source}$$

where G is independent of Z_S. G_A is defined with an assumption that conjugate matching of both source and load depend on Z_S but not Z_L. G_T depends on both Z_S and Z_L. Whenever input and output are both conjugately matched, gain is maximized and $G = G_A = G_T$.

The average power delivered to network

$$P_{in} = \frac{|V_1^+|^2}{2Z_0}(1 - |\Gamma_{\rm in}|^2) = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_S^2|}{|1-\Gamma_S\Gamma_{\rm in}|^2}(1-|\Gamma_{\rm in}|^2)$$

The power delivered to load

$$P_L = \frac{|V_2^-|^2}{2Z_0}(1 - |\Gamma_{\rm L}|^2) = \frac{|V_s|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{\rm L}|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_{\rm L}|^2|1-\Gamma_S\Gamma_{\rm in}|^2}$$

Then, the power gain

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_{\rm L}|^2)}{(1-|\Gamma_{\rm in}|^2)|1-S_{22}\Gamma_{\rm L}|^2}$$

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{\rm out}|^2)}$$

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_{\rm L}|^2)}{|1-\Gamma_S\Gamma_{\rm in}|^2|1-S_{22}\Gamma_{\rm L}|^2}$$

where

$$P_{avs} = P_{in}|_{\Gamma_{\rm in}=\Gamma_S^*} = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_S|^2)}$$

$$P_{avn} = P_L|_{\Gamma_{\rm L}=\Gamma_{\rm out}^*} = \frac{|V_s|^2}{8Z_0}\frac{|S_{21}|^2|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{\rm out}|^2)}$$

If $\Gamma_{\rm L} = \Gamma_S = 0$, then $G_T = |S_{21}|^2$.

If $S_{21} = 0$, then $\Gamma_{\rm in} = S_{11}$, *Unilateral Transducer Power Gain,* G_{TU}

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_{\rm L}|^2)}{|1-\Gamma_S\Gamma_{\rm in}|^2|1-S_{22}\Gamma_{\rm L}|^2}$$

More generally, most useful power definition is Transducer Power Gain account for both source and load mismatch

$$G_T = G_S G_0 G_L$$

where

$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{\rm in}|^2}\ ,\ G_0 = |S_{21}|^2\ ,\ G_L = \frac{1-|\Gamma_{\rm L}|^2}{|1-S_{22}\Gamma_{\rm L}|^2}$$

If transistor is unilateral, $S_{21} = 0$; $\Gamma_{\rm in} = S_{11}$, $\Gamma_{\rm out} = S_{22}$, then

$$G_{TU} = G_T G_0 G_L$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}\ ,\ G_0 = |S_{21}|^2\ ,\ G_L = \frac{1-|\Gamma_{\rm L}|^2}{|1-S_{22}\Gamma_{\rm L}|^2}$$

Similar relation can be obtained by Equivalent circuit parameter.

7.2. Stability

There are necessary conditions for a transistor amplifier to be stable based on the possible oscillation for input and output impedance has a negative real part as a two-sub group:

- *Conditional Stability:* If $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$, network is stable for a range of passive source and load impedance.

- *Unconditional Stability:* If $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive sources and loads, network is unconditionally stable.

The stability condition is usually frequency dependent since matchings generally depend on frequency (stability may be possible for a frequency but not possible for others). Rigorous treatment of stability requires S parameters of network have no poles in the right-half complex plane in addition to $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$. If device is unilateral $S_{21} = 0$, more simply results $|S_{11}| < 1$ and $|S_{22}| < 1$ are enough for stability.

- *Stability Circles*: Applying the above requirement for unconditional stability, following conditions have to be satisfied

$$|\Gamma_{in}| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right| < 1$$

$$|\Gamma_{out}| = \left|S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}\right| < 1$$

These conditions define a range for Γ_S and Γ_L where amplifier will be stable. Finding this range by using Smith chart, plotting the input and output *Stability Circles* are defined as loci in the Γ_L (or Γ_S) plane for which $|\Gamma_{in}| = 1$ (or $|\Gamma_{out}| = 1$), then define boundaries between stable and unstable regions. The equations for input and output stability conditions can be extracted as

$$C_{center}^{input} = \frac{\left(S_{11} - \left[S_{11}S_{22} - S_{12}S_{21}\right]S_{22}^*\right)^*}{|S_{11}|^2 - \left|S_{11}S_{22} - S_{12}S_{21}\right|^2}$$

$$R_{radius}^{input} = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |S_{11}S_{22} - S_{12}S_{21}|^2} \right|$$

$$C_{center}^{output} = \frac{(S_{22} - [S_{11}S_{22} - S_{12}S_{21}]S_{11}^*)^*}{|S_{22}|^2 - |S_{11}S_{22} - S_{12}S_{21}|^2}$$

$$R_{radius}^{output} = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |S_{11}S_{22} - S_{12}S_{21}|^2} \right|$$

If device is unconditionally stable, the stability circles must be completely outside (or totally enclose) the Smith chart. This can be stated mathematically as

$$\big||C_L| - R_L\big| > 1 \quad \text{for} \quad |S_{11}| < 1$$

$$\big||C_L| - R_L\big| > 1 \quad \text{for} \quad |S_{22}| < 1$$

If $|S_{11}| > 1$ or $|S_{22}| > 1$, the amplifier can not be unconditionally stable because a source (or load impedance) leading to $\Gamma_L = 0$ (or $\Gamma_S = 0$) can cause $|\Gamma_{in}| < 1$ (or $|\Gamma_{out}| > 1$). If transistor is only conditionally stable, Γ_S and Γ_L must be chosen in stable regions. Specially unconditionally stability can be tested with the methods of *Rollet's Condition* or *μ Parameter*.

7.3. Single Stage Amplifier Design

Maximum gain with stability can be realized when input and output sections provide a conjugate match between source and load impedance, but generally as a narrowband. To perform this

$$\Gamma_{in} = \Gamma_S^* \;,\; \Gamma_L = \Gamma_{out}^*$$

conditions simultaneously have to be satisfied means that also by maximizing the transducer gain, first of all Γ_S, then Γ_L should be solved by considering stability conditions. It is also preferable to design for less than the maximum obtainable gain, to improve bandwidth (or to obtain a specific amplifier gain). To do that, *Constant Gain Circles* on the Smith chart to represent loci of Γ_S and Γ_L that give fixed values of gain are used. Besides stability and gain, the Noise Figure of the amplifier should be minimized by using *Constant Noise Figure Circles*.

7.4. Broadband Amplifier Design

The bandwidth can be improved with designing for less than maximum gain will improve bandwidth, but the input and output ports will be poorly matched. Common approaches to solve this problem are listed below

- Compensated Matching Network,
- Resistive Matching Network,
- Negative Feedback,
- Balanced Amplifiers,
- Distributed Amplifiers.

7.5. Power Amplifiers

This is used to increase power level with consideration of efficiency, gain, intermodulation and thermal effect. *Amplifier Efficiency* is defined as

$$\eta = \frac{P_{out}}{P_{DC}}$$

with the effect of input power, *Power Added Efficiency, PAE*

$$\eta_{PAE} = \frac{P_{out} - P_{in}}{P_{DC}} = \left(1 - \frac{1}{G}\right)\eta$$

where G is power gain. *PAE* drops quickly with frequency. Another parameter is *Compressed Gain* defined as the gain of amplifier at 1 dB compression gain as

$$G_1(dB) = G_0(dB) - 1$$

where G_0 is small signal (linear) power gain. Class A amplifiers with theoretically maximum efficiency % 50 are inherently linear that transistor is biased to conduct over entire range of input signal cycle (low-noise amplifier). Class B amplifiers with theoretically maximum efficiency % 78 are biased to conduct only during one-half of input signal cycle (Push-Pull amplifier). Class C amplifiers with efficiency near % 100 are operated with transistor near cut-off for more than half of the input signal cycle (in a resonant circuit, constant envelope modulation). Higher classes such as D, E, F and S are also used with high efficiency.

7.5.1. Large Signal Characterization

If input power is small enough, S parameter is independent from input power and linear small signal model is suitable for modeling. But for high input powers, transistor behaves as a nonlinear device (*Large-Signal Characterization*) and more difficult to design. The following methods are possible for Large Signal Characterization

- Measure the output power as a function of source and load impedances and produce tables, then determine large signal source and load reflection coefficients to maximize power gain for a particular output power.

- Plot contours (*Load-Pull Contours*) of constant power output on a Smith chart as a function of load reflection coefficient with conjugately matching at input and design for a specified gain.

- Use nonlinear equivalent of the transistor circuit.

Especially for designing Class A amplifiers, the stability can be checked by using small signal model because instabilities begin at low signal levels.